MTH 202 Section 003 Exam 2

Name: Solutions

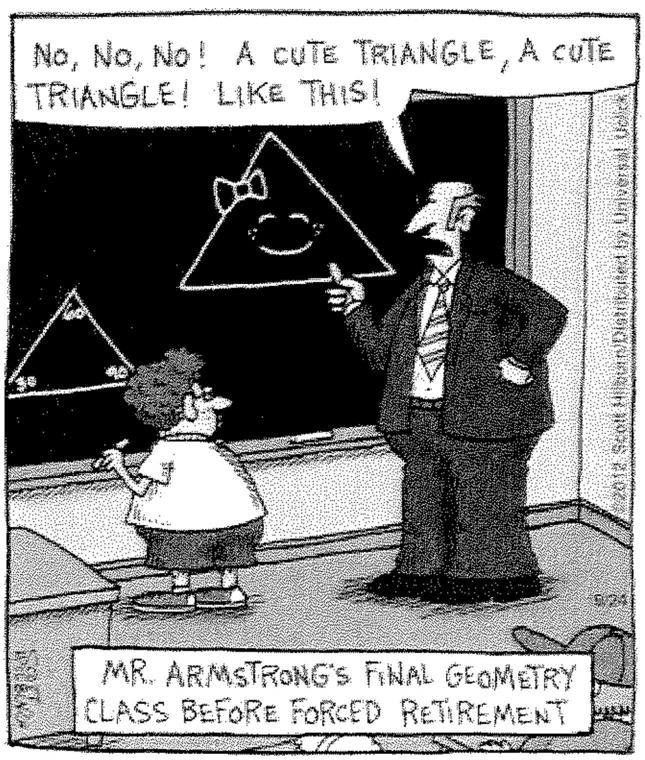
Signature: Solutions

Date: 6 November 15

Do not start this exam until instructed; you will have 50 minutes to finish the exam. No notes, books, calculators, phones or electronic devices are allowed on this exam. If you have a question, raise your hand; otherwise, there is no talking during the exam.

There are 12 problems on this exam on 6 pages, in addition to this cover page. The point values of each problem vary, but are listed in the questions.

Good luck!



From Argyle Sweater.

1. (2+2+2+2=8 points) For the following problems, no work is necessary - just give the answer.

   (a) Describe the teaching sequence for *area.*

   - Count square units
   - Rectangles w/ whole number sides.
   - —— " —— fractional sides.
   - —— " —— real sides.

   (b) Name at least 2 polygons that tessellate.

   Squares

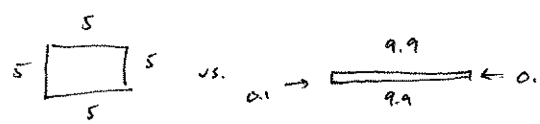
   (Any) triangles.

   (c) 3 pairs of congruent sides proves congruence between triangles (SSS congruence). If, similarly, there are 4 pairs of congruent sides between two *quadrilaterals* are the two figures necessarily congruent?

   No. (See rhombus.

   (d) If a square and a rectangle that is not a square have the same perimeter which has the larger area? *Hint:* Construct an example.

   The square.



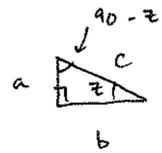
2. (2+2+2+2=8 points) For the following problems, mark true or false. No work is necessary.

All parallelograms with a right angle are *rectangles.*	True False
A triangle with sides of lengths 24, 35, 51 units is a *right* triangle.	True False
A parallelogram with perpendicular diagonals is necessarily a *kite.*	True False
Every number can be written as a fraction.	True False

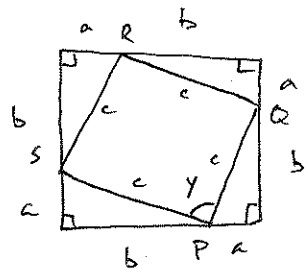
3. (20 points) State and prove the Pythagorean Theorem.

Thm If a and b are the lengths of the legs of a right triangle and c is the length of the hypotenuse, then $a^2 + b^2 = c^2$.

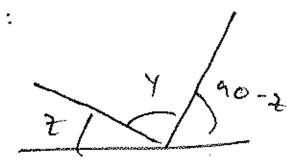
pf.



Construct. $\longrightarrow$



Zoom in at P:



1) PQRS is a square.

Proof: It's a rhombus. Now $y + (90 - z) + z = 180$ by angles on line (at P) gives $y = 90$.

A rhombus with a right angle is a square.

2) Area (PQRS) $= c^2$.

3) Area of big square $= \begin{cases} (a+b)^2 & \text{(side lengths)} \\ c^2 + 4 \cdot \frac{1}{2} ab & \text{(PQRS and 4 triangles} \end{cases}$

4) Compute: $(a+b)^2 = c^2 + 4 \cdot \frac{1}{2} ab$

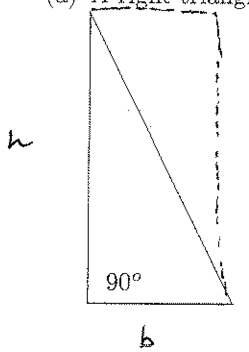
$$a^2 + b^2 + 2ab = c^2 + 2ab$$

$$a^2 + b^2 = c^2.$$

Correction

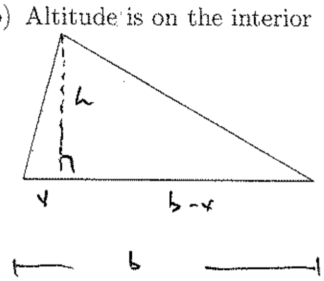
4. (5+7=12 points) ~~Prove~~ Explain why the area of a triangle is $\frac{1}{2}$base $\times$ height in the following cases:

(a) A right triangle



Complete to a rectangle. The two triangles are congruent by SSS. The rectangle has area bh, so each has $\frac{1}{2}$ of that area.

(b) Altitude is on the interior

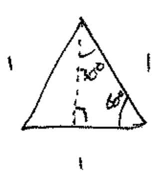


Cut into right triangles. The total area is $\underbrace{\frac{1}{2}xh}_{\text{left}} + \underbrace{\frac{1}{2}(b-x)h}_{\text{right}}$

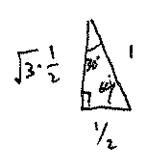
$$= \frac{1}{2}xh + \frac{1}{2}bh - \frac{1}{2}xh$$

$$= \frac{1}{2}bh.$$

5. (10 points) Find the area of an equilateral triangle with sides of length 1 unit.



Cut as shown. Want height h:

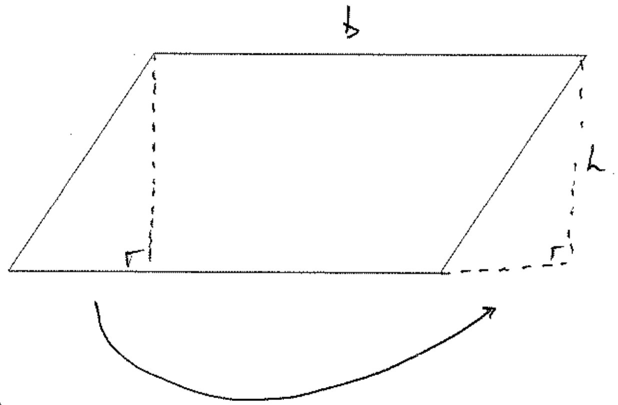


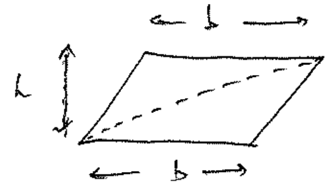
by properties of 30-60-90 Δ.

$$\therefore \quad h = \frac{1}{2}\sqrt{3}$$

$$\therefore \quad \text{Area} = \frac{1}{2}\cdot 1 \cdot \frac{1}{2}\sqrt{3} = \frac{\sqrt{3}}{4}.$$

6. (5 points) Explain how to find the area of the following parallelogram in 2 different ways. Mark any lengths used in your explanation.





2) Cut to two triangles. Each has area $\frac{1}{2}bh$.

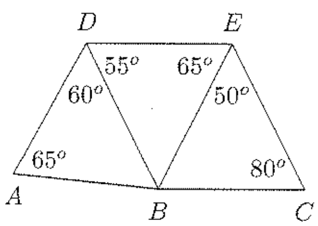
Total $= 2 \cdot \frac{1}{2} bh$

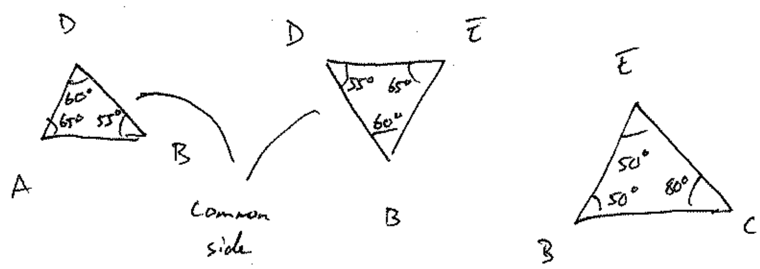
1)

Move this triangle.

Get a rectangle. Area = bh.

7. (5 points) Which of the triangles in the following figure are congruent?





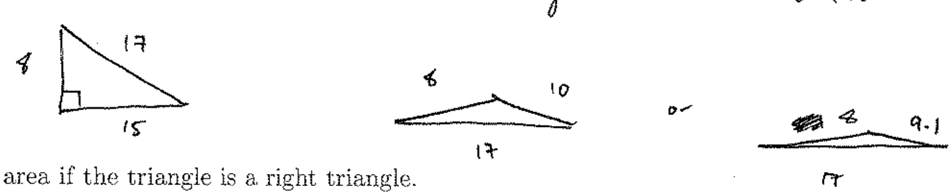
Congruent by ~~SAS~~. ASA.

Not congruent to either.

8. (3+2=5 points) Suppose you know the longest side of a triangle is 17 cm and one leg is 8 cm long.

(a) Explain why the area cannot be found.

Here are two such triangles with different areas:

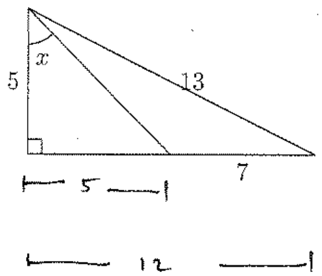


(b) Find the area if the triangle is a right triangle.

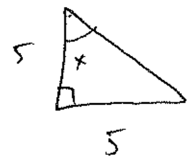
$\frac{1}{2} \cdot (8\text{cm}) \cdot (15\text{ cm})$

$= 4\text{ cm} \cdot 15\text{ cm} = \underline{60\text{ cm}^2}$

9. (5 points) Find the angle x.



$12 - 7 = 5$ by Pythagorean Theorem.

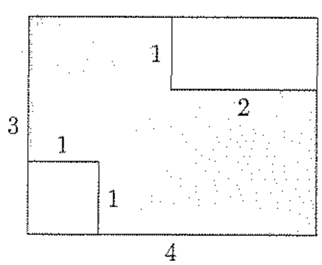


Isosceles right triangle

$\therefore$ 45-45-90 triangle.

$\therefore$ $\underline{\underline{x = 45^\circ}}$

10. (6 points) Find the area of the shaded region. All lengths are given in inches.

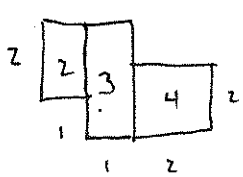


$$\text{Area} = \text{Area (rectangle)} - \text{Area (holes)}$$

$$= (3\text{ in} \times 4\text{ in}) - \big[(1\text{ in}) \times (2\text{ in}) + (1\text{ in}) \times (1\text{ in})\big]$$

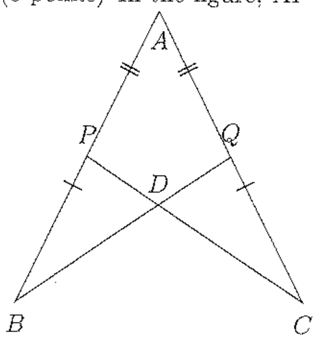
$$= 12\text{ in}^2 - 3\text{ in}^2 = \underline{\underline{9\text{ in}^2}}$$

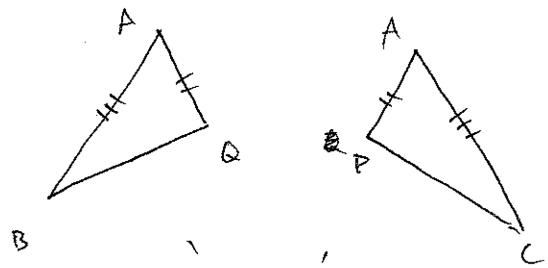
Alternate Solution :



Get $4\text{ in}^2 + 3\text{ in}^2 + 2\text{ in}^2 = \underline{9\text{ in}^2}$.

11. (8 points) In the figure, $AP = AQ$ and $BP = CQ$. Show that $BQ = CP$.



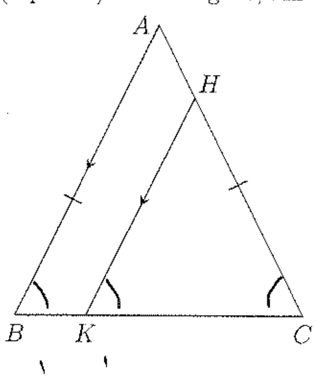


Congruent by SAS. Angle A is common.

Lengths add. { Note $AB = AP + PB$
$= AQ + CQ$
$= AC$, which is why the sides marked /// are congruent.

Now $BQ = CP$ since the segments correspond.

12. (8 points) In the figure, $AB = AC$ and $\overline{AB}$ is parallel to $\overline{HK}$. Show that $HK = HC$.



equal angles by transversal.

$\angle B = \angle K$ since $\overline{BC}$ is transversal of $\overline{AB}$, $\overline{HK}$.

$\angle B = \angle C$ since $\triangle ABC$ is isosceles.

$\therefore$ $\angle C = \angle K$

$\therefore$ $\triangle HKC$ is isosceles

$\therefore$ $HK = HC$.

Recall: Isosceles
$\Updownarrow$
Two equal sides
$\Updownarrow$
Two equal angles.